KIROJO COLLEGE ADVANCED LEVEL PURE MATHEMATCS
REVISION AND EXAMINATION QUESTIONS - SET I
Compiled by: Nkalubo Joseph - 2010-08-20

1. Solve the equation $3 + 2\cos 2x = 4\cos x$ for $\pi \le x \le 2\pi$. (05 Marks)
2. *Solve* the simultaneous equations $16^{x+y} = 4^{x-y}$ and $\log_2(2x+1) - 1 = \log_2 y$. (05 Marks)
3. *U*se the substitution x = sixθ to evaluate $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{x^2\sqrt{1-x^2}}\,dx$ (05 Marks)
4. (i) Find the angle between the planes 3x + 4y + z = 11 and x – y + 2z = 0
   (ii) Obtain the equation of the plane which passes through the points A(2, 3, 1), B(3, -1, 4) and C(6, 2, 0). (05 Marks)
5. Two curves are defined by loci of the points $P(t^{2,} t^3)$ and $Q(3t – 2, t^2 + 4)$; where t is a parameter. Find the value of t at the point of intersection of these curves. Find also the gradient of each curve at this point. (05 Marks)
6. Expand $\frac{(1+x)}{(1-2x)^3}$ in ascending powers of x up to and including the term in x^5. (05 Marks)
7. Given that $y = \log_5^{[2x^3 Sin(2x^2-1)]}$. Find $\frac{dy}{dx}$ and simplify your answer. (05 Marks)
8. (*i*) Find $\int 10^y dy$. (ii) If $\frac{dy}{dt} = \sin x$ and $\frac{dx}{dt} = 1 + \cos x$. Find y in terms of x (02/03 Marks)
9. (a) Given that 2 – 5i is a root of $z^3 – 7z^2 + 41z + k = 0$, find the other roots and the value of the real number k.

(b) Given that $z_1 = 1 + i$, $z_2 = \sqrt{3} - i$, $z_3 = 1 + i\sqrt{3}$. Using De' Moivres's theorem or otherwise simplify $\frac{z_1^2 - z_2}{4z_3}$ giving your answer in modulus - argument form.

10. (a) Solve the equation $\frac{dy}{dx} = \frac{x+y}{x}$
    (*b*) For a particle projected upwards in a resisting medium, the velocity V is given by the equation $\frac{dv}{dt} + kv + g = 0$, where k, g are constants. If the initial velocity is v_0, show that the particle comes to rest afer a time $\frac{1}{k}\log_e\left\{\frac{kv_0 + g}{g}\right\}$
11. (a) If the roots of the equation $ax^2 + bx + c = 0$ are tanα and tanβ, find the value of cos (α+β) in terms of a, b and c
    (b) Given that α and β are roots of the quadratic equation $ax^2 + bx + c = 0$, determine the equation whose roots are α+β and $\alpha^3 + \beta^3$. Hence or otherwise solve the equations; α + β = 2; $\alpha^3 + \beta^3 = 26$.
12. (a) Express $\ln\sqrt{\frac{1 + x}{1 - x}}$ as a series of terms i ascending powers of x up to and including the term x^5. Hence the value of $\ln\sqrt{\frac{11}{3}}$ to six decimal places.
    (b) Show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ and that $\tan^{-1}(2-\sqrt{3}) = \frac{\pi}{12}$
13. (a) Show that the equation on a tangent to the parabola $y^2 = 4ax$ at $P(at^2, 2at)$ is $x – ty + at^2 = 0$.

(b) Find the equation of the tangent to $y^2 = 4ax$ passing through (4a, 5a) and state their points of contact. The tangent of P meets the directrix at Q. Find the loci of the mid – point of PQ.

14. (a) The first term of an arithmetic progression (A.P) is 73 and the ninth is 25. Determine;

(i) The common difference.

(ii) The number of terms that must be added to give a sum of 96.

(b) A geometrical progression (G.P) and an A.P have the same first term. The sums of their first, second and third terms are 6, 10.5 and 18 respectively. Calculate the sum of their fifth terms.

15. (a) Solve the equation $16\sin\Theta\cos\Theta = \tan\Theta + \cot\Theta$ for $0^0 \leq \Theta \leq \pi$. (05 Marks)

(b) Given that $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$; show that,

(i) $\tan\dfrac{\alpha+\beta}{2} = \dfrac{a}{b}$ (ii) $\sin\ \alpha+\beta\ = \dfrac{2ab}{a^2+b^2}$ (02/05 Marks)

16. *Integrate*: (a) $\dfrac{(x-2)^2}{x^3+1}$ (b) $(e^{2x}\cos 3x)$, with respect to x (12 Marks)

17. Determine the turning points including points of inflexion of the curve $y = \dfrac{x}{x^2+1}$ hence sketch it.

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1. (a) Given that $7\tan k + \cot k = 5\sec k$, find all the values of k in the interval between 0^0 and 180^0 inclusive.

(b) The acute angles A and B are such that CosA = ½ and SinB = 1/3. Show without using tables or calculators that

$$\tan(A+B) = \frac{9\sqrt{3}+8\sqrt{2}}{5}$$

2. (a) Determine the possible values of x in the equation $\log_2 x + \log_x 64 = 5$.

(b) Solve the simultaneous equations: $x + 2y - 3z = 0$

$$3x + 3y - z = 5$$

$$x - 2y + 2z = 1.$$

3. (a) The function $f(x) = x^3 + px^2 - 5x + q$ has a factor $(x - 2)$ and has a value of 5 when x = -3. Find p and q.

(b) The roots of the equation $ax^2 + bx + c = 0$ are α and β. Form an equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$

(c) Simplify $\dfrac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form $p + q\sqrt{3}$ where p and q are rational numbers.

4. (a) An inverted cone with vertical angle of 60^0 is collecting water leaking from the tap at a rate of $0.2m^3/s$. If the height of the water collected in the cone is 10cm, find the rate at which surface area is increasing.

(b) (i) $\int(5 - x^2 + \frac{18}{x^4})dx$ (ii) $\int_0^1\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)dx$

5. Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point (2, 4). Find the values of the constants a, b and c.

6. Prove the identity: $2\cot\ A/2\ - \sin A = \cot^2\ A/2\ \sin A$

7. Show that $\int_{\sqrt{3}}^{\infty}\dfrac{dx}{x\sqrt{1+x^2}} = \dfrac{1}{2}\ln 3$

8. Find the eccentricity, the coordinates of the foci and the equation of the directrices of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$.

9. Prove that: $\frac{d}{dx}[\log_2 x+\log_x 2]=\frac{(\ln 2x)(\ln \frac{x}{2})}{(\ln 2^x)(\ln x)^2}$

10. (a) Evaluate: $\int_9^{25}\frac{dx}{\sqrt{x}-\sqrt{x-9}}$ (05 Marks)

(b) The area bound by the curve y = *lnx*, the x – axis and the line x = e is rotated about the x – axis through 360^0. Show that the volume generated is equal to e – 2. (07 Marks)

11. (a) Use small changes to estimate $\sqrt{0.041}$ (06 Marks)

(b) Find the equation of the normal to the curve $y^2(y – 3x) = 3 – x^2$ at the point (-1, 2) (06 Marks)

12. (*a*) Solve the equation: $\sqrt{x - 3}+\sqrt{2x+1}=\sqrt{3x+4}$ (06 Marks)

(b) Given that y = ln(2 - e^x), show that $\frac{d^2y}{dx^2}+\left[\frac{dy}{dx}\right]^2=\frac{dy}{dx}$, hence find the first three non - vanishing terms of the Maclaurine's expansion of y(x) (06 Marks)

13. (i) Find the coordinates of the point of intersection of the line $\frac{x - 2}{3}=\frac{3-y}{2}=z+1$ and the plane 2x + y +3z = 11

(ii) Determine the angle between the line and the plane in (i) above. (06 Marks)

Given the curve $y=\frac{2x^2-8}{2x-5}$;

(i) Find the equations of the asymptotes

(ii) Find the stationary points, hence or otherwise obtain the range of values of y within which the curve does not lie.

(iii) Sketch the curve. (12 Marks)

14. A boy starts to sip a 900ml soda from a bottle at a rate of $10cm^3$ per minute. Given that his rate of consumption is inversely proportional to the square root of the volume of soda remaining at any time, find the time he takes to empty the bottle. (12 Marks)

15. (a) Solve the equation: Sin^3x = six2x for $0^0 \le x \le 360^0$. (06 Marks)

(b) Given that A, B, C are angles of a triangle, show that Sin(A - B) + Cos(A – B)tanC = Sin2BSecC. (06 Marks)

16. A polynomial P(x) leaves a remainder 6 when divided by $x^2 – 4$, and a remainder x + a when divided by $x^2 – x – 2$. Find;

(a) The value of the constant a,

(b) The remainder when P(x) is divided by

(i) $x^2 + 3x + 2$

(ii) $x^3 + x^2 – 4x – 4$.

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1) Solve the equations; 2x + y +3z = 6, 5x – 2y + z = 4, x – y + 4z = 4 (05 Marks)

2) Solve the equation 3 + 2cos2x = 4cox for $\pi \le x\ 2\pi$ (05 Marks)

3) Given that $y = \log_5[2x^3\sin(2x^2 – 1)]$. Find $\frac{dy}{dx}$ and simplify your answer. (05 Marks)

4) A line r passes through the point with position vector – 4*i* + 7*j* + 5*k* and is parallel to vector 3*i* – 2*j* + 4*k*. Find the;

(a) Equation of the line r. (b) Point of intersection of line r and line r = -5*j* – 11*k* + μ(2*i* + *j* + 8*k*).

5) Show that $\int_1^3 \left(\frac{2x^2 + 3x}{2x - 1} \right) dx$ (05 Marks)

6) Given that $2 - 5i$ is a root of $z^3 - 7z^2 + 41z + \lambda = 0$, find the real number λ and the other roots. (05 Marks)

7) Two curves are defined by the loci of the points $P(t^2, t^3)$ and $Q(3t - 2, t^2 + 4)$ where t is a parameter. Find the value of t at the point of intersection of these curves. Find also the gradient of each at this point. (05 Marks)

8) U_1, U_2, U_3, U_4.... are terms in a G.P with U_1 as the first term. Given that $U_1 + U_3 = 26$ and $U_3 + U_6 = 650$, find the possible values of U_4. (05 Marks)

9) (a) Given that Z is complex number such that, $Z = \frac{p}{2 - i} + \frac{q}{1 + 3i}$; where p and q are real numbers. Find the values of p and q if; $\text{Arg}Z = \frac{\pi}{4}$ and $|Z| = 7$.

(b) Show that the locus of the complex number Z, such that $\left| \frac{z - 1 - i}{z - 1 + i} \right| = 2$ is a circle and hence determine the area of this circle. (Take $\pi = \frac{22}{7}$)

10) (a) Solve the equation $y\frac{dy}{dx} = x + 2y$ (05 Marks)

(b) A body is placed in a room which is kept at a constant temperature. The temperature of the body falls at a rate KΘ, where K is a constant and Θ is the difference between the temperature of the body and that of the room at time t.

(i) Form a differential equation containing K, Θ and t and hence show that $\Theta = \Theta_0 e^{-kt}$, where Θ_0 is the temperature difference at t = 0.

(ii) If the temperature of the body falls 5^0C in the first minute and 4^0C in the second minute, show that the fall of temperature in the third minute is 3.2^0C.

11) A curve is given by $y = \frac{(x-1)(x-5)}{(x+1)(x-3)}$

(i) Show that the curve does not have turning points (02 Marks)

(ii) Determine the equation of the asymptotes (02 Marks)

(iii) Sketch the curve. (08 Marks)

12) (a) Find the Cartesian equation of a line passing through the point A(1, 3, 3) and B(2, 5, 6).

(b) Given that line $\frac{x - 3}{4} = \frac{y - 3}{-3} = \frac{z + 3}{4}$ meets the plane $4x - 3y - 4z = 0$ at point M. Find;

(i) The coordinates of point M

(ii) The angle the line makes with the plane

13) (a) Solve for x, $2\text{Cos}2x\text{Cos}x - \text{Cos}x + 1 = 0$ where $0^0 \leq x \leq 180^0$.

(b) Prove that $\left(\frac{1 - \sin 2\theta}{1 + \sin 2\theta} \right)^{\frac{1}{2}} = \frac{1 - \tan\theta}{1 + \tan\theta}$, hence or otherwise solve for θ, if $\left(\frac{1 - \sin 2\theta}{1 + \sin 2\theta} \right)^{\frac{1}{2}} + 2 = 0$. (12Mks)

14) (a) Find the value of n if $3({}^nP_4) = {}^{n-1}P_5$.

(b) From a group of 6 boys and 4 girls of the science club, 5 members are to be selected to represent the club in a science workshop. In how many ways can the selection be done?

(i) If there must be exactly two girls in the science affair?

(ii) If at least one boy and one girl must be in the science affair?

15) (a) Evaluate $\int_2^3 \frac{2x^2+1}{(x+2)(x-1)^2}dx$ correct to 4 dps. (b) If y = $\tan^{-1}\left(\frac{1-x}{1+x}\right)$, show that $\frac{dy}{dx} = -\frac{1}{1+x^2}$.

16) (a) Show that if the equations $x^2 + 2x + n = 0$ and $x^2 + kx + 3 = 0$ have a common root, then $(3 - n)^2 = (6 - nk)(k - 2)$.

(c) Prove that if x is too small that its cube and higher powers are neglected, $\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{x^2}{2}$. By taking x = $\frac{1}{9}$, prove that $\sqrt{5} = \frac{181}{81}$

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1. Express $5 - 2x - 3x^2$ in the form $a - b(x + c)^2$. Deduce the maximum or minimum value of the expression.

2. Differentiate from first principles: $y = \sin x^2$. (05 Marks)

3. Evaluate: $\int_0^{\frac{\pi}{6}} \text{Cos}5x\text{Cos}2x\,dx$ (05 Marks)

4. A(1, 3) and B(4, 6). P(x, y) is a variable point which moves in such a way that $(\overline{AP})^2 + (\overline{PB})^2 = 34$. Show that the locus of P describes a circle. Find the centre and radius of the circle. (05 Marks)

5. Solve for x and y if; $\log_9(x - y) = \log_3(x + y)$ and $x^2 - y^2 = 8$ (05 Marks)

6. Given that y = $\sqrt{\frac{1+\text{Sin}x}{1-\text{Sin}x}}$, show that $\frac{dy}{dx} = \frac{1}{1 - Sinx}$ (05 Marks)

7. Show that the line $x - 2y + 10 = 0$ is a tangent to the ellipse $\frac{x^2}{64} + \frac{y^2}{9} = 1$ (05 Marks)

8. Express $\sqrt{\frac{\sin 2\theta - \text{Cos}2\theta - 1}{2 - 2Sin2\theta}}$ in terms of $\tan\theta$ (05 Marks)

9. (a) A geometrical sequence has first term 16 and common ratio ¾. If the sum of the first n terms exceeds 60, find the possible value of n. (05 Marks)
   (b) The sum of the first *p* terms of an arithmetic progression is *q* and the sum of the first *q* terms is *p*. Show that the sum of the first (*p* + *q)* terms is –(*p* + *q)*. (07 Marks)

10. (a) Solve the equations $x + y = 2$ and $x^3 + y^3 = 26$ simultaneously (04 Marks)
    (b) If α and β are the roots of $x^2 + px + q = 0$, express $(\alpha - \beta^2)(\beta - \alpha^2)$ in terms of p and q. Hence, deduce that for one root to be square of the other, then, $p^3 - 3pq + q^2 + q = 0$. (05 Marks)
    (c) Rationalise: $\frac{2\sqrt{2}}{3\sqrt{2}+4}$ (03 Marks)

11. (a) If y $\frac{3\sin 2x + 4\cos 2x}{2x+1}$, show that $(2x + 1)\frac{dy}{dx} + 2y = 10Cos(2x + \alpha)$ (06 Marks)
    (b) Solve the equation $e^{2x} - 4e^x + 3 = 0$ (05 Marks)

12. Expand $\sqrt{\frac{1+5x}{1-5x}}$, as far as and including the term in x^3. Taking the first three terms and $x = \frac{1}{9}$, evaluate $\sqrt{14}$ correct to 4 significant figures (12 Marks)

13. (a) If A + B + C = 180^0, prove that sinA + sinB + sinC = 4Cos½ACos½BCos½C. (06 Marks)
(b) The acceleration of a particle after time *t* seconds is given by a = 5 + cos½t. If initially the particle is moving at $1ms^{-1}$, find its velocity after 2π seconds and the distance it would have covered by then. (06 Marks)

14. (a) Find the equation of a circle with centre (1, 2), which touches the line 3x – 4y + 10 = 0. (04 Marks)
(b) The curve $b^2x^2 + a^2y^2 = a^2b^2$ intersects the positive x – axis at A, and the positive y – axis at B.
(i) Determine the equation of the perpendicular bisector of AB.
(ii) Given that this line intersects the x – axis at P and that M is the bisection of AB. Find the area of triangle PMA. (08 Marks)

15. (a) Prove that $\tan^{-1}\frac{q}{p+q} + \tan^{-1}\frac{p}{p+2q} = \frac{\pi}{4}$ (05 Marks)
(b) Express 7cos*x* + 24sin*x* in the form RCos(x – α) and state the maximum and minimum values of 7cos*x* + 24sin*x*, hence, find the values of x for $-180^0 \le x \le 180^0$ for 7cos*x* + 24sin*x* = 10. (7 Marks)

16. (*a*) Differentiate w.r.t.x: (i) $y = 2e^{\cos x}$ (ii) $y = \frac{e^{\sin x}}{\tan^{-1} x}$ (b) Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}dx$. (12 Marks)

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1) Evaluate $\int_1^2 \frac{x^3}{1+x^2}dx$ to 3 decimal places (05 Marks)

2) Without using tables, show that $\frac{\log\sqrt{27}+\log\sqrt{8}-\log\sqrt{125}}{\log 6-\log 5} = \frac{3}{2}$ (05 Marks)

3) Solve for x in, (*i*) $\log_5^x + \log_x^5 = \frac{3}{2}$ (ii) $\log_5^{(2-x)} = \log_{25}^{(5-4x)}$ (05 Marks)

4) Evaluate $\int_0^1 x\sqrt{4-3x^2}\,dx$ (05 Marks)

5) Given that $\log_3 x = p$ and $\log_{18} x = q$, show that $\log_6 3 = \frac{p}{p-q}$ (05 Marks)

6) Work out (*i*) $\int_3^4 \frac{1}{x^2-3x+2}dx$ (ii) Show that $\int_0^{\frac{\pi}{2}} \frac{Cos^4 x}{1+\sin x}dx = \frac{1}{12}\ 3\pi - 4$ (05/08 Marks)

7) Solve for x, $\sqrt{(x-1)} + 2\sqrt{(x-4)} = 4$ (05 Marks)

8) *If* $\log_a^n = x$, $\log_c^n = y$ for $n \neq 1$, prove that $\frac{x-y}{x+y} = \frac{\log_b^c - \log_b^a}{\log_b^c + \log_b^a}$ (05 Marks)

9) If $5^x.25^{2y} = 1$ and $3^{5x}.9^y = 1/9$. Determine the values of x and y. (05 Marks)

10) (a) Eliminate Θ from x = SinΘ, y = CosΘ (02 Marks)
(b) Find the real value of x satisfying $e^x + e^{-x} = 4$. (03 Marks)
(c) Simplify $2\log_a b \times 3\log_b a$ (ii) Find the value of a such that $\int_0^a (x^2+2x-6)dx = 0$ (05 Marks)

11) (a) Find the Cartesian equation of a line passing through the points A(2, -3, 4) and B(3, -7, 12) (03 Marks)
(b) Given that the line in (a) above meets the plane 4x + 5y – 2z = 5 at point M. Find;
(i) The coordinates of point M (ii) The angle between the line and the plane. (09 Marks)

12) (a)The expression $6x^2 + x + 7$ leaves the same remainder of 72 when divided by (x + a) and (x + 2a), where a is not zero, determine the value of a. (04 Marks)

(b) Given that α and β are the roots of the equation $x^2 + bx + c = 0$.

(i) Show that $(\alpha^2 + 1)(\beta^2 + 1) = (c - 1)^2 + b^2 = 0$. (04 Marks)

(iii) Find in terms of b and c the quadratic equation whose roots are $\frac{\alpha}{\alpha^2+1}$ and $\frac{\beta}{\beta^2+1}$ (04 Marks)

13) Sketch the curve y = (3 – x)(x + 1)(x + 3) clearly stating and distinguishing all the turning points (12 Marks)

14) (a) An inverted right circular cone of semi – vertical angle 45^0 is collecting water from a gutter at a steady rate of $18\pi cm^3s^{-1}$. When the depth of water is *h*cm, the rate at which the depth is rising is ½ cms^{-1}. Determine the value of h (05 Marks)

(b) An inverted right circular cone of vertical angle 120^0 is collecting water from a tap at a steady rate of $18\pi cm^3min^{-1}$, Find; (i) the depth after of water 12 min (ii) the rate of increase of depth at this instant (05/02).

15) (a) Find the perpendicular distance from the point A, with position vector $\begin{pmatrix}4\\-3\\10\end{pmatrix}$ to the line l with vector equation, $r = \begin{pmatrix}\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$ (04 Marks)

(b) Solve x, y and z if $\frac{y+z}{5} = \frac{z+x}{8} = \frac{x+y}{9}$ and 6 (x + y + z) =11 (06 Marks)

(c) Write Sin*x* + Sin*3x* in the form pCos*x*Sin*kx* and state the values of p and k. (02 Marks)

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1) Solve the simultaneous equations;

$$xy + x + y = 23$$
$$xz + x + z = 41$$
$$yz + y + z = 27$$

(05 Marks)

2) The point P(14+2λ, 5+2λ, 2-λ) lies on a fixed straight line for all values of λ. Find the Cartesian equation for this line and find the Cosine of the acute angle between this line and the plane x – y = 0. (05 Marks)

3) Evaluate: $\int_0^{\pi} t^3 \sin t^2 dt$ (05 Marks)

4) A circle with centre P and radius r touches externally both circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 8 = 0$, prove that the x-coordinate of p is $\frac{r}{3} + 2$ (05 Marks)

5) A curve is defined parametrically by the equation; x = a(2t + sin2t) and y = a(1 – cos2t). Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t and simplify your answer. (05 Marks)

6) Expand $(1+x)^{\frac{1}{4}}$ in ascending powers of x up to the forth term using binomial theorem hence estimate $\sqrt[4]{16.5}$ correct to three decimal places. (05 Marks)

7) Find the value of x between 0^0 and 360^0 for which sin*x* + sin*3x* = cos*x* + cos*3x*. (05 Marks)

8) Given that $Z_1 = \frac{11+12i}{3-4i}$ and $Z_2 = 2-5i$. Find the distance between Z_1 and Z_2. (08 Marks)

9) Given that $y = \frac{\sin x - 2\sin 2x + \sin 3x}{\sin x + 2\sin 2x + \sin 3x}$

(a) Prove that $y = -\tan^2 \frac{x}{2}$ (04 Marks)

(b) Hence find the exact value of $\tan^2 15$ in the form $p + q\sqrt{r}$, *where* p, q and r are integers (04 Marks)

(c) Find the value of x between 0^0 and 360^0 for which $2y + \sec^2 \frac{x}{2} = 0$. (04 Marks)

10) (a) Sand falls on to a horizontal ground at a rate of $9m^3$ per minute and forms a heap in the shape of a right circular cone with vertical angle $120^{0.}$ show that $20\sqrt{3}$ seconds after the sand begins to fall, the rate at which the radius of the base of the pile is increasing is $\frac{\sqrt{3}}{\pi^{\frac{2}{3}}} m$ per minute. (07 Marks)

(b) Using small changes find the approximate value of $\sqrt[5]{33}$ (05 Marks)

11) (a) An arithmetic progression (A.P) contains n terms. The first term is 2 and the common difference is $\frac{2}{3}$. If the sum of the last four terms is 72 more than the sum of the first four terms, find n. (06 Marks)

(b) A sequence of numbers is formed by adding together corresponding terms of an A.P and a G.P whose common ration is 2. The first term of the sequence is 57, the second term is 94 and the third term is 171, find the fourth term. (06 Marks)

12) (a) *Express* the function $f(x) = \frac{x+2}{(x+1)(2x-1)}$ as a sum of partial fractions and hence $\int f(x)dx$. (07 Marks)

(b) Show that the area enclosed by the curve $y = e^x$, the y- axis and the line y = 2 is given by $2\left(\ln 2 - \frac{1}{2}\right)$

13) (a) The complex number Z satisfies $\frac{Z}{Z+2} = 2 - i$. Find the real and imaginary parts of Z and the modulus and argument of Z (07 Marks)

(b)Find the locus defined by the $\arg\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{2}$. (05 Marks)

14) Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$. The straight line $4x - 9y + 8a = 0$ meets the parabola at the points A and B. If the normals to the parabola at A and B meet at R. Find the coordinates of R and verify that it lies on the parabola. (12 Marks)

15) (a) Find;

(i) The Cartesian equation of the line joining the points (3, 1, 0) and (-1, 2, 2).

(ii) The coordinates where this line intersects the plane $x + 2y - z = 7$

(b) Two lines are given by $r_1 = i + 6j + 3k + t(2i - j + k)$ and $r_2 = 3i + 3j + 8k + s(i + k)$

(i) Calculate the angle between these lines

(ii) Show that the two lines do not meet. (12 Marks)

16) (a) Solve the differential equation $\sin x \frac{dy}{dx} + 2y\cos x = 1$. (04 Marks)

(b) An electric kettle switches off itself when the temperature of water in it reaches 100^0C. At 11:00am, Mr. Ali came back; he measured the temperature of water and found it to be 65^0C. 20 minutes later, he measured it again and found it to be 45^0C. According to Newton's law of cooling, the rate of cooling of the body in air is proportional to the excess temperature over the surrounding at any time, t. If the surrounding temperature was 25^0C, Mr. Ali wants to know the time when the kettle switched itself off. (08 Marks)

*****END*****

1. Solve for x: $\log_6 36x + \log_{36} 6x = 6$ (05 Marks)
2. Solve for θ, $2\cos^2\theta = 3 - 3\sin\theta$ such that $0^0 \leq \theta \leq 180^0$ (05 Marks)
3. If the roots of $3x^2 + x + 2 = 0$ are α and β, find an equation with integral coefficients whose roots are $1 + \frac{1}{\alpha^2}$ and $1 + \frac{1}{\beta^2}$ (05 Marks)
4. Given that $y = e^{\text{Sinx}}$; find, $\frac{d^2y}{dx^2}$ (05 Marks)
5. *If* $x = t^2 - t$ and $y = 3t + 4$, find the value of $\frac{dy}{dx}$ when x = 2 and the equation of the line at that point.
6. Evaluate $\int_1^{\sqrt{2}} (x + \tan x)dx$ to 4 decimal places. (05 Marks)
7. Find the equation of the line through (1, 2, 3) and (3, 3, 4). (05 Marks)
8. Define the locus described by $\overline{AP}^2 + \overline{PB}^2 = 64$ where A(1, 3) and B(4, 6). (05 Marks)
9. (a)(i) Differentiate with x^{Sinx} (ii) x^{1-x} (03/03 Marks)

   (b) The distance, x in metres covered by a particle after time, t in seconds is given as $x = e^{-t}\cos t$. Determine the velocity and acceleration of the particle at the instant when $t = \frac{\pi}{4}$ (03/03 Marks)
10. (a) The polynomial $x^3 + Ax - 12$ has a root (x – 3), find the value of A hence determine the other factors of the polynomial. (02/03 Marks)

    (b) Express $3x^2 + 12x + 5$ in the form p(x + q) + 1 and state the values of p, q and r. Hence find the maximum value of $\frac{1}{3x^2 + 12x + 5}$ (07 Marks)
11. *Evaluate*: (a) $\int_0^{\frac{\pi}{2}} x\sin^2 x dx$ (b) $\int_0^2 \frac{3x^2 - 7x + 6}{(x-3)(x+1)}dx$ (05/07 Marks)
12. (a) Determine the possible values of Z if; $Z\overline{Z} + 2iZ = 12 + 6i$ (07 Marks)

    (b) Describe and hence sketch the locus defined by $\left|\frac{Z-1}{Z+1}\right| = 2$ (05 Marks)
13. (a) Find the equation of the circle passing through A(1, 3), B(4, -5) and C(9, -1) and hence state its centre and radius. (06 Marks)

    (b) If the tangent to the circle at A meets the y – axis at (0, n) and the x – axis at (m, 0), find the values of m and n (take the gradient of the tangent to zero decimal places) (06 Marks)
14. (a) Show that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ (06 Marks)

    (b) Given that $\sin 3\theta$ = p and $\sin^2\theta = \frac{3}{4} - q$, show that $p^2 + 16q^3 = 12q^2$ (06 Marks)
15. (a) *Find* the point of intersection of the line $\frac{x - 2}{3} = \frac{y-3}{-2} = z + 1$ and the plane 2x + y + 3z = 11 (07Mks)

(b) Determine the angle between the plane and the line in (a) above. (05 Marks)

16. (a) Use calculus of small changes to estimate $\sqrt{98}$ (06 Marks)

(b) *Use* binomial expansion to find the first four terms of $(4-3x)^{-\frac{1}{2}}$, hence estimate $\frac{1}{\sqrt{3.97}}$. (06 Marks)

*****END*****

1. Find the values of and b if; the expression b + ax – $4x^2$ + $8x^3$ leaves a remainder of 10 when divided by (x + 1) and it is divisible by (x – 1) (04 Marks)
2. Given that $\log_2 x + 2\log_4 y = 4$, show that xy = 16. Hence solve for x and y in the simultaneous; equations $\log_{10}(x+y) = 1$, $\log_2 x + 2\log_4 y = 4$ (06 Marks)
3. Determine the value of x between 0^0 and 360^0 inclusive for which secx = 5 – $\tan^2$x. (05 Marks)
4. Prove that $8^n - 7n + 6$ is divisible for all positive integers (05 Marks)
5. Find the locus of the complex number Z given that $Arg(Z+2) = \frac{3\pi}{4}$ (04 Marks)
6. Two lines have the vector equations $r_1 = \begin{pmatrix}3\\-1\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ *and* $r_2 = \begin{pmatrix}4\\4\\1\end{pmatrix} + \mu\begin{pmatrix}-1\\1\\2\end{pmatrix}$ find the position of the point of intersection of the two lines. (05 Marks)
7. Find the perpendicular distance from the point A, position vector $\begin{pmatrix}4\\-3\\10\end{pmatrix}$ to the line, L with vector equation $r = \begin{pmatrix}1\\2\\3\end{pmatrix} + \mu\begin{pmatrix}3\\-1\\2\end{pmatrix}$ (06 Marks)
8. Find the range or ranges, of values of K can take for $Kx^2 - 2xK - 3K - 12 = 0$ to have real roots. (05 Marks)
9. (a) *Given* that $\int_0^a (x^2+2x-6)dx = 0$, find the value of a (04 Marks)

   (b) Differentiate with respect to x: $\frac{3x+4}{\sqrt{2x^2+3x-2}}$ (05 Marks)

   (c) Given that the vectors *ai – 2j + k* and *2ai + aj – 4k* are perpendicular, find the value of a. (03 Marks)
10. The position vectors of points P and Q are 2*i – 3j + 4k* and *3i – 7j + 12k* respectively.
    (a) Find the equation of the line which passes through points P and Q.
    (b) Given that PQ meets the plane 4x + 5y – 2z = 5 at S. , find;
    (i) The coordinates of point S
    (ii) The angle between PQ and the plane. (12 Marks)
11. (a) Show that Sin3*x* = 3Sin*x* – 4$\sin^3 x$. Hence solve the equation Sin3*x* + Sin*x* = 0 for $0^0 \le x \le 360^0$ (06 Marks)

    (b) Solve 3Sin*x* – Cos*x* = 3 for $0^0 \le x \le 360^0$. (06 Marks)
12. (a) In the expansion of $(1 + ax)^n$, the first three terms are $1-\frac{5x}{2}+\frac{75x^2}{8}$. Find the values of n and a and state the values of x for which the expansion is valid. (06 Marks)

    (b) Expand $(1 + x)^{1/2}$ in ascending powers of x as far as the term in x^2 and hence find the approximation for $\sqrt{1.08}$. Deduce that $\sqrt{12} \approx 3.464$ (06 Marks)

13. (a) Show that part of the line 3y = x + 5 is a chord to the circle $x^2 + y^2 - 6x - 2y - 15 = 0$ and find the length of the chord (06 Marks)

(b) Find the equation of the circle which passes through the points (0, 1), (4, 3) and (1, -1). (06 Marks)

14. (a) Prove by induction that $(\cos x + i\sin x)^n = \cos nx + i\sin nx$ hence solve $Z^3 - 1 = 0$ (07 Marks)

(b) Express $i(1 + i)^4$ in modulus – Argument form (05 Marks)

1. Solve the equation Sin2x + 2Cos2x = 2 for $0^0 \leq x \leq 360^0$ (05 Marks)
2. The normal through (0, 0) to the circle $x^2 + y^2 - 6x - 8y + 21 = 0$ meets the circle at A. Find the coordinates of A.
3. Given that y = Sin e^t and t = *ln*Sin*x*, show that $\frac{dy}{dx} = \sqrt{1-y^2}\cos x$ (05 Marks)
4. Find the coordinates point of intersection of the lines $r = \begin{pmatrix}-2\\-8\\9\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$ *and* $r = \begin{pmatrix}7\\-4\\1\end{pmatrix} + \mu\begin{pmatrix}3\\-1\\-1\end{pmatrix}$ (05 Marks)
5. (*a*) *Find* $\int 10^y dy$ (b) If $\frac{dy}{dt} = \sin x$ and $\frac{dx}{dt} = 1 + \cos x$. Find y in terms of x. (05 Marks)
6. Show that $\int_{\ln 4}^{\ln 9} \sqrt{e^x}\, dx = 2$. (05 Marks)
7. Solve the equation $2\sec^2 x = 1 + 3\tan x$ for $0 \leq x \leq 2\pi$. (05 Marks)
8. The polynomial P(x) is divided by x + 2 and x – 2, the remainders are 4 and 2 respectively. Find the remainder when P(x) is divided by $x^2 - 4$. (05 Marks)
9. A business pays an insurance premium of Sh.5 million per annum with the understanding that it can withdraw its money with 4% compound interest per annum when the need arises. How much will withdraw at the end of tenth years? (05 Marks)
10. Prove by induction that: $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$ (05 Marks)
11. Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} Co\sec 2x Cotx\, dx = \frac{1}{2}$ (05 Marks)
12. Show that the circles $(x-1)^2 + (y+3)^2 = 64$ and $(x+2)^2 + (y-1)^2 = 9$ touch each other, and find the coordinates of their point of contact. (05 Marks)
13. (a) Given that x is so small that its cube and higher powers can be neglected, find the first three terms of the expansion of $\sqrt{\frac{1+x}{1-x}}$. By putting $x = \frac{1}{17}$, show that $\sqrt{2} \approx \frac{1226}{867}$.

    (b) The eighth term of arithmetic progression (AP) is twice the fourth term and the sum of the first eight terms is 30. Find (i) The first four terms of the AP (ii) The sum of the first four terms of the AP. (12 Marks)

14. (a) Find the greatest and least values of the expression below and the values of x for which they occur where $0^0 \leq x \leq 360^0$, $\frac{5}{\sqrt{6}\sin x - \sqrt{3}\cos x + 5}$.

    (b) Show that $\frac{\sin 3\phi \sin 6\phi + \sin\phi \sin 2\phi}{\sin 3\phi \cos 6\phi + \sin\phi\cos 2\phi} = \tan 5\phi$

(c) If $\tan\phi + \sin\phi = x$ and $\tan\phi - \sin\phi = y$, prove that $(x^2 - y^2)^2 = 16xy$ (12 Marks)

15. (a) Find the component of the vector 5i – 4j + 8k in the direction of vector 2i – j + 2k.

(b) Given that the angle between the line $r = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and r = $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + m\begin{pmatrix} 1 \\ \lambda \\ 1 \end{pmatrix}$ is 60^0, find the;

(i) Values of λ. (ii) Point of intersection of the lines (12 Marks)

16. *Sketch* the curve: $y = \dfrac{2(x^2 - x - 2)}{x^2 + x - 12}$ (12 Marks)

17. (a) Find the equation of the tangents to the parabola $y^2 = 9x$ which passes through the point A(4, 10).

(b)(i) Show that the equation of the line joining the points P(cp, c/p) and Q(cq, c/q) on the rectangular hyperbola $xy=c^2$ is x + pqy = c(p + q). Hence deduce the equation of the tangent of the point P.

(ii) The normal to the rectangular hyperbola xy = 8 at the point B(4, 2) meets the asymptotes at M and N. Find the length of MN. (12 Marks)

*****END*****

1. a) (i) Prove that $\dfrac{Sinx + Sin4x + Sin7x}{Cosx + Cos4x + Cos7x} = \tan 4x$ (05 Marks)

(ii) Show that $Sin2x = \dfrac{\tan x}{1 + \tan^2 x}$ (05 Marks)

b) Evaluate $\log_{25}^{125}$ without using tables or calculators (02 Marks)

2. a) If $Cosb + Cosd = p$ and $Sinb - Sind = q$, show that $Cos(b-d) = \dfrac{p^2 - q^2}{p^2 + q^2}$ (04 Marks)

b) Solve the equation $C\text{os}^{-1}(x) + S\text{in}^{-1}(x\sqrt{8}) = \dfrac{\pi}{2}$ (05 Marks)

c) Given that $\log_3^x = p$ and $\log_{18}^x = q$, show that $\log_6^3 = \dfrac{q}{p-q}$ (03 Marks)

3. (a) $\int Cos^3 3x dx$ (3 Marks) (b) Determine $\dfrac{d}{dx}\left(\ln \dfrac{x}{\sqrt{(1+x^2)}}\right)$ when x = 2 (04 Marks)

(b) $\int \cos^{-1}\theta d\theta$ (05 Marks)

4. Solve the simultaneous equations; $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2 \\ 5x + 4y + 2z &= 3 \end{aligned}$ (05 Marks)

5. Show that if the equations $x^2 + ax + 1 = 0$ and $x^2 + x + b = 0$ have a common root, then $(b-1)^2 = (a-1)(1-ab)$ (05 Marks)

6. Solve the equation $\dfrac{x^2 + 4x}{3} + \dfrac{84}{x^2 + 4x} = 11$ (05 Marks)

7. Differentiate $y = \sqrt{x}$ from first principles (05 Marks)

8. (a) Solve for x in $\log_5^x + \log_x^5 = 3$ (05 Marks)

(b) Given that $y = \sqrt{3x^2 + 2}$, show that $2\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right) = 6$ (07Marks)

9. (a) Given that the equation $2x^2+5x-8=0$ has roots α and β, find the equation whose roots are $\frac{1}{(\alpha+2)^2}$ *and* $\frac{1}{(\beta+2)^2}$ (06 Marks)

(b)When $x^2 + px + q$ is divided by x - h, the remainder is the same as when the quadratic expression is divided by x - 2h. Find the two possible values of h (06 Marks)

10. (a) The straight line x-y-6=0 cuts the curve $y^2=8x$ at P and Q. Calculate the length PQ. (06 Marks)

(b) Prove that the points (1, -1), (-1, 1), (√3, √3) are the vertices of an equilateral triangle. Hence find the coordinates of the orthocentre of the triangle. (06 Marks)

11. (a) A particle moves in a straight line and its distance **S**(m) from the point at which it is situated at zero time is given in terms of the time **t**(s) by the formula, $\mathbf{S = 45t + 11t^2 - t^3}$. Find the **velocity** and **acceleration** after 3 seconds and prove that the particle will come to rest after 9 seconds (08 Marks).

(b)A particle moves along a straight line so that after **t** seconds, its distance from the origin, a fixed point on the line is given by $\mathbf{S = t^3 - 3t^2 + 2t}$. When is the particle at the origin? (04 Marks)

3. Solve simultaneously $x^2 + xy + y^2 = 3$, $x^2 + 2xy + 2y^2 = 5$ (05 Marks)

4. Given that $2x = t + t^{-1}, 2y = t - t^{-1}$, show that $\frac{d^2y}{dx^2} = -\frac{8t^3}{(t^2-1)^3}$ (05 Marks)

5. Given that $\tan\alpha = \frac{a\sqrt{3}}{2b-a}$ and $\tan\beta = \frac{2a-b}{b\sqrt{3}}$, find the values of $\alpha - \beta$ between 0^0 and 360^0 (05 Marks)

6. Find the point of intersection of the line $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4} = t$, and the plane $3x + 4y + 2z = 25$ (05 Marks)

7. Solve the equation $\frac{dy}{dx} = \frac{y}{2x+1}$, given that x = 4 when y = 6, determine the value of x when y = 5 (05Marks)

8. A curve with a stationary point at (-1, 2) has its second derivative as 6x+2. find the equation of the curve (05Marks)

9. Solve the equation *sinx* + *cos2x* =1 for $-\pi \le x \le \pi$ (05Marks)

10. Find $\int \frac{5}{\sqrt{x}\,(2+\sqrt{x})^3}dx$ (05 Marks)

11. (i) The first three terms of a G.P are t-3, 2t-4 and 4t-3 in that order. Find the value of t and s_8 to 2 decimal points (06Marks)

(ii) Solve by row reducing to Echelon form, for the values of x, y and z if
$$2x + y - 3z = 3$$
$$x - 3y - 5z = 1$$
$$6x - 2y + z = 9$$
(06Marks)

12. (a) Solve the differential equation $x\frac{dy}{dx} = y + 2$, give y(3) = 7 (05Marks)

(b) The rate of growth of bacteria in a container is proportional to the number of bacteria present, if initially there were 100 bacteria and after 2hours they were 300; find,

(i) The number of bacteria after 3 hours (04Marks)

(ii) The time it will take the bacteria to be 500 (03Marks)

13. (a) Prove that $\tan\frac{x+y}{2}-\tan\frac{x-y}{2}=\frac{2\sin y}{Cosx+Cosy}$ (06Marks)

(b) Solve $Cosx+Sin2x=Cos3x$ for $0\le x\le 2\pi$ (06Marks)

14. (i) Integrate $\int_0^{\frac{\pi}{2}}\frac{Sinx}{3Sinx+4Cosx}dx$ (06Marks)

15. Solve for x in, $3^{2x} - 12 \times 3^{x+1} + 243 = 0$ (05 Marks)

16. If y = $\frac{1}{2x+3}$ and Δx, Δy are the corresponding changes in x and y respectively; find $\frac{\Delta y}{\Delta x}$ in terms of Δx and Δy. Hence deduce $\frac{dy}{dx}$ for the above function. (05 Marks)

17. Show that $Cot\ 2x=\frac{1-\tan^2 x}{2\tan x}$. Hence find the value of x such that $0^0 \le x \le 360^0$ for which $2-2\tan x=\frac{1-\tan^2 x}{2\tan x}$. (05 Marks)

18. Simplify $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form p + q$\sqrt{3}$. (b) Given that $\sqrt{3}=1.732,$ find $\sqrt{0.0675}$ *without* using a calculator.

19. *Without* using a calculator, estimate the value of $\sqrt[3]{8.02}$ using calculus of small changes (05 Marks)

20. The expression $b + ax - 4x^2 + 8x^3$ leaves a remainder of 10 when divided by (x+1) and it is divisible by (x-1) Find the value of a and b. (05 Marks)

21. Find the value of x in $2\sqrt{x}+\sqrt{2x+1}=7$ (05 Marks)

22. Simplify $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form p + q$\sqrt{3}$. (b) Given that $\sqrt{3}=1.732,$ find $\sqrt{0.0675}$ *without* using a calculator.

Find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x=\frac{2t}{1+t^2}$ and $y=\frac{1-t^2}{1+t^2}$ (05 Marks)

23. (a) An arithmetic progression (A.P) contains n terms. The first term is 2 and the common difference is 2/3. If the sum of the last four terms is 72 more than the sum of the first four terms, find n. (06 Marks)

(c) Express $5 - 2x - 3x^2$ in the form $a - b(x + c)^2$. Deduce the maximum or minimum value of the expression. (06 Marks)

24. (a) Solve the equation Cosx + Cos2x = 1 for values of x from 0^0 to 360^0 inclusive.

(b) Prove that $\frac{CosA+CosB}{SinA+SinB}=Cot\left(\frac{A+B}{2}\right)$ hence deduce that $\frac{CosA+CosB}{SinA+SinB}=\tan\left(\frac{C}{2}\right)$ where A, B and C are angles of a triangle. (12 Marks)